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SOLVED PYQ

Paper: Introductory Statistics for Economics

Course: B. A.(Hons.) Economics I Year

Year: 2021

Modified as Per New NEP Syllabus

ECON003: Introductory Statistics for Economics
Course : B. A.(Hons.) Economics I Year
Duration : 3 Hours **Maximum Marks : 90**

SEMESTER–1
2021

Attempt any one from questions 1 and 2.

SECTION A

Q. 1. A sample was collected of the daily earnings of a food vendor on a street outside a college for 30 working days. The distribution of earnings is as follows:

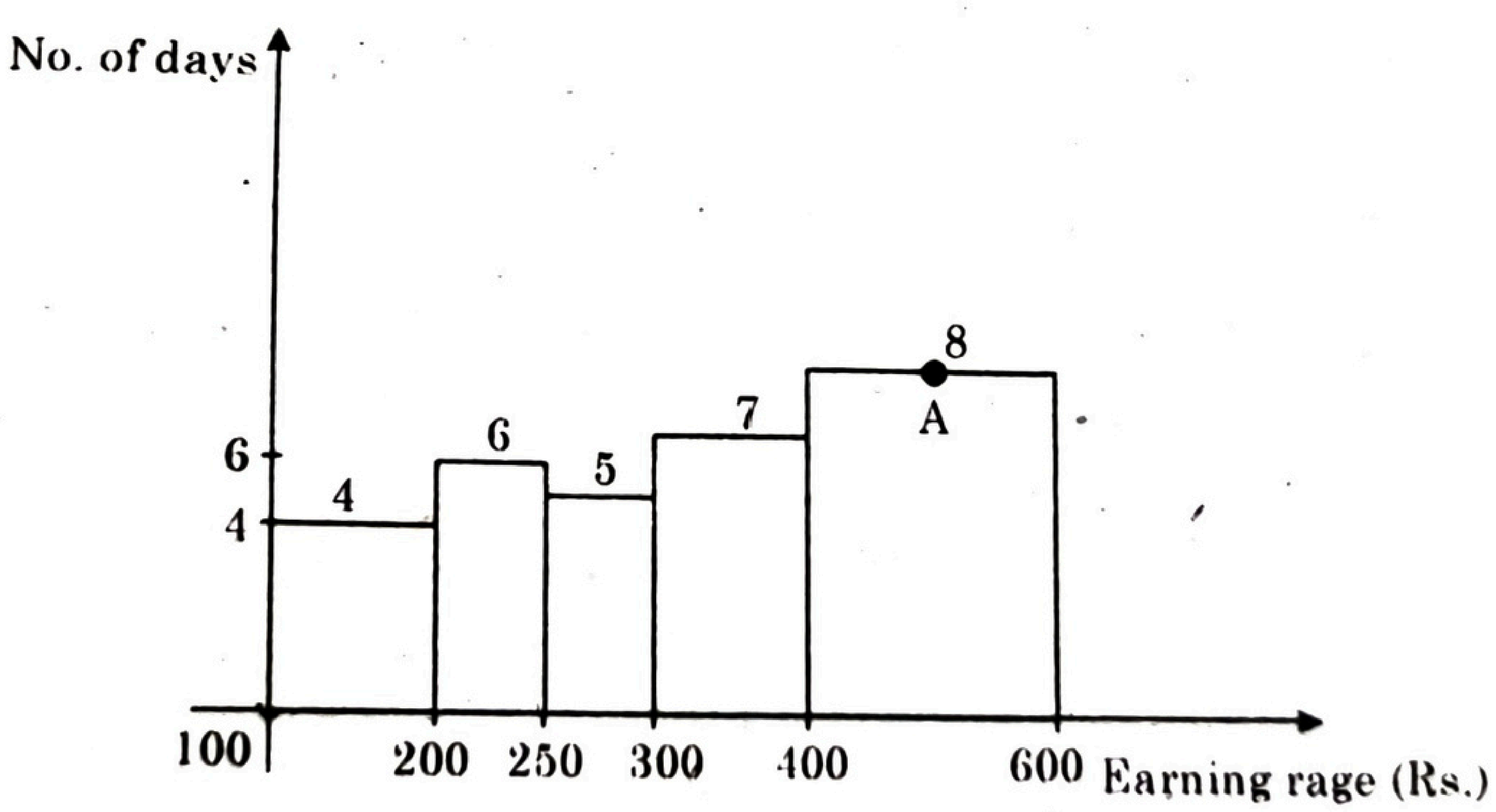
Earning Range (Rs.)	100-200	200-250	250-300	300-400	400-600
No. of days:	4	6	5	7	8

(i) Draw a histogram to depict the data and describe its shape.

(ii) Mark on the histogram where you expect the mode and the median to lie.

(iii) If the municipal authorities were to charge Rs. 50 per day from the food vendor, what would be the impact on the mean and standard deviation of the vendor's earnings? **4,2,4**

Ans. (i)



The data seems to be negatively skewed.

(ii) The mode is the value that appears most often in a set.

In the histogram, it would be somewhere in the last bar of the histogram, like point A.

(iii) This would mean the earning was X_i and now it is Y_i, hence

$$Y_i = X_i - 50$$

$$\Rightarrow \quad E(Y_i) = E(X_i) - 50$$

so mean would decrease by ₹ 50

and $$Var(Y_i) = Var(X_i - 50)$$

$$\Rightarrow \quad Var(Y_i) = Var(X_i)$$

$$\Rightarrow \qquad \sqrt{Var(Y_i)} = \sqrt{Var(X_i)}$$

Hence, the standard deviation does not change.

Q. 2. The wheat yield (X in quintals per acre) for a farm in Punjab for a sample of 8 years was 50, 56,47, 27, 52,43,46 and 55.

(i) Compute the sample mean, the 10% trimmed mean and the sample standard deviation for the wheat yield.

(ii) Is the sample mean the better measure of location for the given data? Why or why not?

(iii) A new hybrid seed promises a yield (Y) given by y_i =1.2x, –8 What is the expected mean, 10% trimmed mean and standard deviation of yield for this hybrid seed? **5,2,3**

Ans. (i)
$$E(X) = \frac{50+56+47+27+52+43+46+55}{8}$$

Arranging the data in the ascending order.

27, 43, 46, 47, 50, 52, 55, 56

$$n = 8, \; n \times \frac{80}{100} = 0.8 \approx 1$$

Leaving first and last term, the trimmed mean

$$= \frac{43+46+47+50+52+55}{6}$$

$$\text{Sample standard deviation} = \sqrt{\frac{\sum_{i=1}^{8}(X_i - \bar{X}.)}{n-1}} = 8.6313$$

(ii) Yes, since there are no extreme values as well as sample mean is always an unbiased estimater of population mean.

(iii)
$$y_i = 1.2\,x_i - 8$$
$$E(y_i) = 1.2\,E(X) - 8$$

Also, the trimmed mean

$$E(\tilde{Y}) = 1.2E(\tilde{X}) - 8$$

Where $\tilde{Y}$, $\tilde{X}$ represent trimmed data.

SECTION B

Attempt *any two* from questions 3,4 and 5.

Q. 3. (a) A coin is tossed. If it comes heads, the coin is tossed one more time. Otherwise, the coin is tossed two more times. All the outcomes are recorded,

(i) Write down the sample space,

(ii) What is the probability of heads appearing twice?

(iii) Are the events "heads appearing once" and "heads appearing twice" independent?

(b) Prove the following for two events A and B with positive probabilities:

(i) If P(A) = 1/3 and P(not B) = 1/4, then A and B are not mutually exclusive.

(ii) If P(AA/B) > P(A), then P(B/A) > P(B). 6, 4

Ans. (a) (i) Sample space = {HH, HT, THH, THT, TTH, TTT}

(ii) $P\text{ (heads appearing twice)} = \frac{2}{6}$

$$= \frac{1}{3}$$

(iii) $P\text{ (heads appearing once)} = \frac{3}{6}$

$$= \frac{1}{2}$$

P (heads appearing once ∩ heads appearing twice)

$$= 0$$

Since P (heads appearing once ∩ heads appearing twice)

≠ P (heads appearing once). P (heads appearing twice)

Hence, the outcomes are not independent.

(b) (i) $P(A) = \frac{1}{3}$

$$P(B') = \frac{1}{4} \Rightarrow P(B) = \frac{3}{4}$$

Suppose they are mutually exclusive,

Hence $P(A \cap B) = 0$

and $P(A) + P(B) = P(A \cup B)$

$$\Rightarrow \quad \frac{1}{3} + \frac{3}{4} = P(A \cup B)$$

$$\Rightarrow \quad P(A \cup B) = \frac{4+9}{12}$$

$$\Rightarrow \quad P(A \cup B) = \frac{13}{12} > 1 \text{ which is not possible}$$

Hence, $P(A \cup B) > 0$ and hence, A and B are not mutually exclusive.

(ii) $P(A/B) > P(A)$

$\Rightarrow \quad \dfrac{P(A \cap B)}{P(B)} > P(A)$

$\Rightarrow \quad \dfrac{P(B \cap A)}{P(A)} > P(B)$

$\Rightarrow \quad P(B/A) > P(B)$

Q. 4. (a) Prove the following:

(i) If A and B are independent events then the events not A and not B are independent,

(ii) If P(not A) = α and P(not B) = β then $P(A \cap B) \geq 1 - \alpha - \beta$.

(b) Suppose it is known that the proportion of people in a town suffering from tuberculosis is 0.001. A test for the disease has the following properties : If a person suffers from the disease, the test correctly identifies it with a probability of 0.99; if a person does not suffer from the disease the test wrongly identifies it with a probability of 0,02. If a randomly selected person tests positive for the disease, what is the probability that the person actually suffers from the disease?

6,4

Ans. (a) (i) $P(A' \cap B') = P(A) . P(B)$ ...(i)

{Since A and B are independent}

$P(A' \cap B') = P(A \cup B)'$

$\Rightarrow \quad P(A' \cap B') = 1 - P(A \cup B)$

$\Rightarrow \quad P(A' \cap B') = 1 - [P(A) + P(B) - P(A \cap B)]$

$\Rightarrow \quad P(A' \cap B') = 1 - [P(A) + P(B) - P(A) P(B)]$ { from (i)}

$\Rightarrow \quad P(A' \cap B') = 1 - P(A) - P(B) [1 - P(A)]$

$\Rightarrow \quad P(A' \cap B') = (1 - P(B)) (1 - P(A))$

$\Rightarrow \quad P(A' \cap B') = P(B') P(A')$

Hence, A′ and B′ are independent

(ii) $P(A') = \alpha \Rightarrow P(A) = 1 - \alpha$

$P(B') = \beta \Rightarrow P(B) = 1 - \beta$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \quad P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$\Rightarrow \quad P(A \cap B) = (1 - \alpha) + (1 - \beta) - P(A \cup B)$

$\Rightarrow \quad P(A \cup B) = 2 - \alpha - \beta - P(A \cap B)$

Now, $P(A \cup B) \leq 1$

Hence, $2 - \alpha - \beta - P(A \cap B) \leq 1$

$\Rightarrow \quad -P(A \cap B) \leq 1 - 2 + \alpha + \beta$

$\Rightarrow \quad -P(A \cap B) \leq \alpha + \beta - 1$

$\Rightarrow \quad P(A \cap B) \geq 1 - \alpha - \beta$

(b) A : Someone suffering from TB

B : Identifies the disease positively

$$P(A) = 0.001,\ P(A') = 0.999$$

$$P\left(\frac{B}{A}\right) = 0.99,\ P\left(\frac{B}{A'}\right) = 0.02$$

$$P\left(\frac{A}{B}\right) = \frac{P(A)\cdot P\left(\frac{B}{A}\right)}{P(A)\cdot P\left(\frac{B}{A}\right) + P(A')\cdot P\left(\frac{B}{A'}\right)}$$

$$= \frac{0.001 \times 0.99}{0.001 \times 0.99 + 0.999 \times 0.02} = 0.04721$$

Q. 5. (a) A box contains 10 balls of which 4 are black. A number n is selected randomly from the numbers 1,2,3,4,5 and a sample of n balls is drawn. What is the probability that all the balls drawn are black?

(b) If events A and B are independent with P(A) = 1/3 and P(not B) = 1/4, find P(A ∪ B).

(c) The probability that a child born to a couple is female is 0.5. What is the probability?

(i) that the third child is a boy given that the first two are girls?

(ii) that the first three children are all boys?

(iii) that at least one of the first three children is a boy?

Ans. (a)

10 balls

6 balls are non-black (B′) | 4 balls are black (B)

If $n = 1$, sample space = {B, B′}

If $n = 2$, sample space = {BB, B′B′, B′B, B′B′}

Likewise

$$P\text{ (All balls are black)} = \frac{1}{5}\times\frac{1}{2} + \frac{1}{5}\times\frac{1}{4} + \frac{1}{5}\times\frac{1}{8} + \frac{1}{5}\times\frac{1}{16} + \frac{1}{5}\times 0$$

$$= \frac{1}{5}\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}\right)$$

$$= \frac{1}{5} \times \frac{8+4+2+1}{16}$$

$$= \frac{3}{16}$$

(b)
$$P(A) = \frac{1}{3}$$
$$P(B') = \frac{1}{4} \Rightarrow P(B) = \frac{3}{4}$$
$$P(A \cap B) = P(A)\,P(B) \quad \{\text{since A and B are independent}\}$$

Now,
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{1}{3} + \frac{3}{4} - \frac{1}{3} \times \frac{3}{4}$$
$$= \frac{13}{12} - \frac{1}{4}$$
$$= \frac{13-3}{12}$$
$$= \frac{10}{12}$$
$$= \frac{5}{6}$$

(c) (i) P (third child is a boy) = 0.5

(ii) P (first three children are boys) = 0.5 × 0.5 × 0.5
= 0.125

(iii) P (atleast one boy) = 1 – P (all girls)
= 1 – 0.125
= 0.875

SECTION C

Attempt question 6 and any two from questions 7, 8 and 9.

Q. 6. The probability density function of a continuous random variable X is

$$f(x) = \frac{6}{4}\left(1 - x^2\right), 0 \le x \le 1.$$ **Find E(8X) and Var (8X).** **2,3**

Ans.
$$E(X) = \int_0^1 x \cdot \frac{6}{4}\left(1 - x^2\right)dx$$
$$= \frac{3}{2}\int_0^1 (x - x^2)dx$$
$$= \frac{3}{2}\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1$$

$$= \frac{3}{2}\left[\frac{1}{2} - \frac{1}{4}\right]$$

$$= \frac{3}{2} \times \frac{1}{4}$$

$$= \frac{3}{8}$$

So, $E(8X) = 8E(X)$

$$= 8 \times \frac{3}{8}$$

$$= 3$$

Now, $Var(X) = E(X^2) - (E(X))^2$

$$= \int_0^1 x^2 \frac{6}{4}(1-x^2)dx - \left(\frac{3}{8}\right)^2$$

$$= \frac{3}{2}\left[\frac{x^3}{3} - \frac{x^5}{5}\right]_0^1 - \frac{9}{64}$$

$$= \frac{3}{2} \times \frac{2}{15} - \frac{9}{64}$$

$$= \frac{1}{5} - \frac{9}{64}$$

$$= \frac{64-45}{320}$$

$$= \frac{19}{320}$$

Now $Var(8X) = 64\ Var(X)$

$$= 64 \times \frac{19}{320}$$

$$= \frac{19}{5}$$

Q. 7. (a) An academic seminar is going to be organized in the coming semester. 5% of the students of college P will qualify for paper presentation in this seminar. Assuming the validity of this premise,

(i) Among 25 randomly selected students from college P, what is the probability that 3,4 or 5 students will qualify for paper presentations?

(ii) What are the expected mean and standard deviation of the number of students from college P who would qualify for paper presentations from a random sample of 100 students taken from college P ? **6,4**

Ans. (a) (i) The required probability

$$P(X = 3) + P(X = 4) + P(X = 5)$$

$$= {}^{25}C_3 (0.05)^3 (0.95)^{22}$$

$$+ {}^{25}C_4 (0.05)^4 (0.95)^{21}$$

$$+ {}^{25}C_5 (0.05)^5 (0.95)^{20}$$

Where X represents the number of students who qualify for paper presentations.

Using the table A.1

$$= p(x \le 5) - p(x \le 2)$$
$$= 0.999 - 0.873 = 0.126$$

(ii) X : same as defined before.

Now $X \sim \text{Bin}(100, 0.05)$ $n = 100$, $p = 0.05$

So,
$$E(X) = np = 100 \times 0.05 = 5$$
$$\text{Var}(X) = npq = 100 \times 0.05 \times 0.95 = 4.75$$

Hence, $\text{s. d}(X) = \sqrt{4.75}$

(b) Suppose *X* is a Poisson random variable. Derive its variance. **6,4**

Ans. $X \sim \text{Pois}(\lambda)$

and
$$P(X = x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, \ldots..$$

Now
$$\text{Var}(X) = \sum_{x=0}^{\infty} x^2 \frac{\lambda^x e^{-\lambda}}{x!} - \left(\sum_{x=0}^{\infty} \frac{x\lambda^x e^{-\lambda}}{x!}\right)$$

Now
$$\sum_{x=0}^{\infty} \frac{x^2 \lambda^x e^{-\lambda}}{x!} = \sum_{x=0}^{\infty} \frac{x\lambda^x e^{-\lambda}}{(x-1)!}$$

$$\Rightarrow \quad \sum_{x=1}^{\infty} x \frac{e^{-\lambda}\lambda^x}{(x-1)!}$$

Let $x - 1 = y$

So, $$\sum_{x=0}^{\infty} \frac{x^2 \lambda^x e^{-\lambda}}{x!} = \sum_{y=0}^{\infty} \frac{y e^{-\lambda} \lambda^{y+1}}{y!} + \sum_{y=0}^{\infty} \frac{e^{-\lambda} \lambda^{y+1}}{y!}$$

$$= \underbrace{\lambda \sum_{y=0}^{\infty} \frac{y e^{-\lambda} \lambda^{y}}{y!}}_{I} + \underbrace{\lambda \,.\sum_{y=0}^{\infty} \frac{e^{-\lambda} \lambda^{y}}{y!}}_{II}$$

Solving II $$\lambda e^{-\lambda} \sum_{y=0}^{\infty} \frac{\lambda^y}{y!} = \lambda e^{-\lambda}\left[1 + \frac{\lambda}{1!} + \frac{\lambda}{2!} + ...\right]$$

$$= \lambda e^{-\lambda} e^{\lambda} \qquad \left\{\text{using } e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + ...\right\}$$

$\Rightarrow$ $$\lambda e^{-\lambda} \sum_{y=0}^{\infty} \frac{\lambda^y}{y!} = \lambda$$

Solving I

$$\lambda e^{-\lambda} \sum_{y=0}^{\infty} \frac{y\lambda^y}{y!} = \lambda e^{-\lambda} \sum_{y=1}^{\infty} \frac{y\lambda^y}{y!}$$

$$= \lambda e^{-\lambda} \sum_{y=1}^{\infty} \frac{\lambda^y}{(y-1)!}$$

$$= \lambda e^{-\lambda}\left[\frac{\lambda}{0!} + \frac{\lambda^2}{1!} + \frac{\lambda^3}{2!} + ...\right]$$

$$= \lambda^2 e^{-\lambda}\left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{1!} + + ...\right]$$

$$= \lambda^2 e^{-\lambda} \cdot e^{\lambda}$$

$$= \lambda^2$$

Hence, $$\sum_{x=0}^{\infty} \frac{x^2 \lambda^x e^{-\lambda}}{x!} = \lambda^2 + \lambda$$

Now $$\sum_{x=0}^{\infty} \frac{x \lambda^x e^{-\lambda}}{x!} = e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^x}{(x-1)!}$$

$$= e^{-l} \lambda e^{l} = \lambda$$

Hence, $$\text{Var}(x) = \lambda^2 + \lambda - \lambda^2 = \lambda$$

Q. 8. (a) Two fair six-sided dice are tossed. The absolute difference in the outcomes of these two tosses is called X. Derive the probability mass function for X.

(b) The probability that there is at least one error while uploading observations on Income by A is 0.2 and for B and C they are 0.25 and 0.4 respectively. If A, B, and C uploaded 90, 136 and 200 observations on income respectively. Find the expected number of correct observations in all. 4,2,4

Ans. (a)

X	Events
0	(1,1), (2,2),(3,3),(4,4),(5,5),(6,6)
1	(1,2),(2,1),(3,4),(4,3),(5,6),(6,5),(2,3).(3,2),(4,5),(5,4)
2	(1,3),(3,1),(2,4),(4,2),(5,3),(3,5),(6,4),(4,6)
3	(1,4),(2,5),(3,6),(4,1),(5,2),(6,3)
4	(1,5),(2,6),(5,1),(6,20
5	(1,6),(6,1)

X	P(X)
0	6/36
1	10/36
2	8/36
3	6/36
4	4/36
5	2/36

(b) Let X be the number of correct observation uploaded by 'A'

Now X~ Bin (90, P)

Q. 9. A car company offers different payment options for car purchasers. For a randomly selected purchaser, let X be the number of months between successive payments. The cumulative distribution function of X is

$$f(x) = \begin{cases} 0 & , \quad x < 1 \\ 0.30 & , \quad 1 \le x < 3 \\ 0.40 & , \quad 3 \le x < 4 \\ 0.45 & , \quad 4 \le x < 6 \\ 0.60 & , \quad 6 \le x < 12 \\ 1 & , \quad \text{otherwise} \end{cases}$$

(a) Compute P $(3 \leq X \leq 6)$ and P $(X \geq 4)$.

(b) What is the probability mass function of X?

(c) Calculate E (X). 4,4,2

Ans. (a)

$$P(3 \leq X \leq 6) = 0.10 + 0.05 = 0.15$$

$$P(X \geq 4) = 0.05 + 0.15 + 0.40 = 0.6$$

(b) Probability Mass function

$$f(x) = \begin{cases} 0 & x < 1 \\ 0.30 & 1 \leq x < 3 \\ 0.10 & 3 \leq x < 4 \\ 0.05 & 4 \leq x < 6 \\ 0.15 & 6 \leq x < 12 \\ 0.40 & 12 \leq x \end{cases}$$

(c)

$$E(X) = \int_1^3 x(0.30)dx + \int_3^4 0.10x\, dx + \int_4^6 0.05x\, dx + \int_6^{12} 0.15x\, dx + \int_{12}^{\infty} 0.40x\, dx$$

Since the last term approaches to infinity

$$E(X) = \propto$$

SECTION D

Attempt any two from questions 10,11 and 12

Q. 10. Given the discrete joint probability function

$$f(x, y) = \frac{x(1+3y^2)}{k}, x = 0, 1, 2\ y = 0, 1$$

(i) For what value of k is $f(x,y)$ a valid probability function?

(ii) Find $E(Y)$

(iii) Find the variance of Y

(iv) What is the conditional distribution of Y when $X = 0$?

(v) Are X and Y independent random variables? 2×5

Ans. (i) $$\sum_x \sum_y \frac{x(1+3y^2)}{k} = 1$$

$$\Rightarrow \quad \frac{1}{k}\left[\sum_x \left(x(1)+x(4)\right)\right] = 1$$

$$\Rightarrow \quad \frac{1}{k}\left[\sum_x (5x)\right] = 1$$

$$\Rightarrow \quad \frac{1}{k}[5+10] = 1$$

$$\Rightarrow \quad \frac{1}{k}(15) = 1$$

$$\Rightarrow \quad k = 15$$

(ii)
$$f(y) = \left(\frac{1+3y^2}{15}\right)+2\left(\frac{1+3y^2}{15}\right)$$

$$\Rightarrow \quad f(y) = \frac{1+3y^2}{5}$$

So,
$$E(y) = 0\left(\frac{1+3(0)^2}{5}\right)+1\left(\frac{1+3(1)^2}{5}\right)$$

$$= \frac{4}{5}$$

(iii)
$$\text{Var}(y) = E(y^2)-(E(y))^2$$

$$= 0^2\left(\frac{1+3(0)^2}{5}\right)+1^2\left(\frac{1+3(1)^2}{5}\right)-\left(\frac{4}{5}\right)^2$$

$$= \frac{4}{5}-\frac{16}{25} = \frac{20-16}{25} = \frac{4}{25}$$

(iv)
$$f(x, y) = \frac{x\left(1+3y^2\right)}{15}$$

$$\Rightarrow \quad f(x) = \frac{x\left(1+3(0)^2\right)}{15}+\frac{x\left(1+3(1)^2\right)}{15}$$

$$= \frac{x}{15}+\frac{4x}{15}$$

$$\Rightarrow \qquad f(x) = \frac{x}{3}$$

and
$$f(y) = \frac{(1+3y^2)}{5}$$

Now
$$f(x) \cdot f(y) = \frac{x}{3}\left(\frac{1+3y^2}{5}\right)$$

$$= \frac{x(1+3y^2)}{15}$$

$$\Rightarrow \qquad f(x) \cdot f(y) = f(x, y)$$

Hence, X and Y are independent

Q. 11. Given the continuous joint probability function

$$f(x, y) = \frac{k(x+y)}{3}, 0 \le x \le, 0 \le y \le 1,$$

compute the following:

(i) For what value of k is $f(x,y)$ a valid probability function?

(ii) Expected value of Y when X= 0.5

(iii) P (X < 0.6,Y < 0.8)

(iv) Marginal distribution of X

(v) Covariance of X and Y

2×5

Ans. (i) $\int_{x=0}^{x=1}\int_{y=0}^{y=1} \frac{k(x+y)}{3} dy\, dx = 1$

$$\Rightarrow \qquad \frac{k}{3}\int_{x=0}^{x=1}\left[xy + \frac{y^2}{2}\right]_{y=0}^{y=1} dx = 1$$

$$\Rightarrow \qquad \frac{k}{3}\left[\int_{x=0}^{x=1}\left(x + \frac{1}{2}\right)\right]dx = 1$$

$$\Rightarrow \qquad \frac{k}{3}\left[\frac{x^2}{2} + \frac{1}{2}x\right]_0^1 = 1$$

$$\Rightarrow \qquad \frac{k}{3}\left[\frac{1}{2} + \frac{1}{2}\right] = 1$$

$\Rightarrow \qquad k = 3$

(ii)
$$f(x, y) = (x + y)$$

$$f\left(y/x = 0.5\right) = (y + 0.5)$$

$$E\left(y/x = 0.5\right) = \int_{y=0}^{y=1} y(y+0.5)\,dy$$

$$= \left[\frac{y^3}{3} + \frac{0.5y^2}{2}\right]_{y=0}^{y=1}$$

$$= \frac{1}{3} + \frac{1}{4}$$

$$= \frac{7}{12}$$

(iii)
$$P(X < 0.6, Y < 0.8) = \int_{x=0}^{x=0.6}\int_{y=0}^{y=0.8} (x+y)\,dy\,dx$$

$$= \int_{x=0}^{x=0.6}\left(\frac{x}{y} + \frac{y^2}{2}\right)_{y=0}^{y=0.8} dx$$

$$= \int_{x=0}^{x=0.6}\left(0.8x + \frac{0.64}{2}\right)dx$$

$$= \left[0.8\frac{x^2}{2} + 0.32x\right]_{x=0}^{x=0.6}$$

$$= \frac{0.8(0.6)^2}{2} + 0.32 \times 0.6$$

$$= 0.4 \times 0.36 + 0.32 \times 0.6 = 0.336$$

(iv)
$$f(x) = \int_{y=0}^{y=1} (x+y)\,dy$$

$$= \left[xy + \frac{y^2}{2}\right]_{y=0}^{y=1}$$

$\Rightarrow \quad f(x) = x + \frac{1}{2}$

(v) $\quad \text{Cov.}(x, y) = E(xy) - E(x)E(y)$

$$E(xy) = \int_{x=0}^{x=1}\int_{y=0}^{y=1} xy(x+y)dydx$$

$$= \int_{x=0}^{x=1} x\left[\int_{y=0}^{y=1}(xy+y^2)dy\right]dx$$

$$= \int_{x=0}^{x=1} x\left[\frac{xy^2}{2}+\frac{y^3}{3}\right]_{y=0}^{y=1} dx$$

$$\Rightarrow \quad = \int_{x=0}^{x=1} x\left(\frac{x}{2}+\frac{1}{3}\right)dx$$

$$= \int_{x=0}^{x=1} x\left(\frac{x^2}{2}+\frac{x}{3}\right)dx$$

$$= \left(\frac{x^3}{6}+\frac{x^2}{6}\right)_{x=0}^{x=1} = \frac{2}{6} = \frac{1}{3}$$

$$E(x) = \int_{x=0}^{x=1} x\left(x+\frac{1}{2}\right)dx$$

$$= \int_{x=0}^{x=1}\left(x^2+\frac{1}{2}x\right)dx$$

$$= \frac{1}{3}+\frac{1}{4}$$

$$= \frac{7}{12}$$

Similarly, $E(y) = \frac{7}{12}$

Hence, $\text{Cov}(x, y) = \frac{1}{3} - \frac{7}{12} \times \frac{7}{12}$

$$= \frac{48-49}{144}$$

$$= -\frac{1}{144}$$

Q. 12. (a) If the covariance between random variables X and Y is 0.6, compute the coefficient of correlation between 5X and 10 – 3Y.

(b) "A value of the coefficient of correlation close to zero is sufficient to conclude that there is no relationship between the two variables." Comment.

(c) The equation for cost (C) of a firm is given by C = 10W + 5X where W is the wage paid to workers and X is the quantity of goods produced. W and X are random variables with means 400 and 50 respectively, and variances 36 and 9 respectively and coefficient of correlation - 0.4. Compute the mean and variance of the firm's cost (C).

2.2.6

Ans. (a) $\text{Cov}(x, y) = 0.6$

$$\text{Cov}(5x, 10-34) = \text{Cov}(5X, 10) - \text{Cov}(5X, 3Y)$$
$$= 0 - 15\,\text{Cov}(x, y)$$
$$= -15 \times 0.6$$
$$= -9$$

(b) No, this means there is an absence of linear relationship between t two

(c)

$$C = 10W + 5X$$
$$E(C) = 10\,E(W) + 5\,E(X)$$
$$= 10 \times 400 + 5 \times 50$$
$$= 4000 + 250$$
$$= 4250$$
$$\text{Var}(C) = 10^2\,\text{Var}(W) + 25\,\text{Var}(X) + 2\,\text{Cov}(X, W)$$
$$= 100 \times 36 + 25(9) + 2(-0.4)\,6 \times 3$$
$$= 3600 + 225 - 0.8 \times 18$$
$$= 3825 - 14.4$$

AGS-S